

Discrete Mathematics

Recitation Course 2

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2-1

Sets

2-1 Ex.8

- Determine whether these statements are true or false.
 - a) $\emptyset \in \{\emptyset\}$ true
 - b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ true
 - c) $\{\emptyset\} \in \{\emptyset\}$ false
 - d) $\{\emptyset\} \in \{\{\emptyset\}\}$ true
 - e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ true
 - f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ true
 - g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ false, 2 sets are equal

Cardinality

- 2-1 Ex.18

What is the cardinality of each of these sets?

– a) \emptyset 0

– b) $\{\emptyset\}$ 1

– c) $\{\emptyset, \{\emptyset\}\}$ 2

– d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$ 3

Power Set

- 2-1 Ex.22

Determine whether each of these sets is the power set of a set, where a and b are distinct elements

- a) \emptyset x
- b) $\{\emptyset, \{a\}\}$ {a}
- c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ x
- d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ {a, b}

Cartesian Products

- 2-1 Ex.32

Explain why $(A \times B) \times (C \times D)$ and $A \times (B \times C) \times D$ are not the same

- The first is a pair, and the second is a triple

- What about $A \times \emptyset$?

- The Cartesian products $A \times B$ and $B \times A$ are not equal, unless $A = \emptyset$ or $B = \emptyset$ (so that $A \times B = \emptyset$) or $A = B$

Cartesian Products (cont'd)

- 2-1 Ex.36

Suppose that $A \times B = \emptyset$, where A and B are sets, what can you conclude?

- We conclude that $A = \emptyset$ or $B = \emptyset$
- To prove this, suppose that neither A nor B were empty
- Then there would be elements $a \in A$ or $b \in B$
- This would give at least one element, namely (a, b) in $A \times B$, so $A \times B$ would not be the empty set
- This contradiction shows that either A or B (or both, it goes without saying) is empty

2-2

Set Operations

2-2 Ex.4

- Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find
 - a) $A \cup B$ $\{a, b, c, d, e, f, g, h\} = B$
 - b) $A \cap B$ $\{a, b, c, d, e\} = A$
 - c) $A - B$ \emptyset
 - d) $B - A$ $\{f, g, h\}$

Mutual Subsets

- 2-2 Ex.20
- Show that if A and B are sets, then $(A \cap B) \cup (A \cap \bar{B}) = A$.
- $A \subseteq (A \cap B) \cup (A \cap \bar{B})$: every element $x \in A$ is an element of either $A \cap B$ (if $x \in B$) or $A \cap \bar{B}$ (if $x \notin B$).
- If $x \in (A \cap B) \cup (A \cap \bar{B})$, then either $x \in A \cap B$ or $x \in A \cap \bar{B}$. In either case, $x \in A$.

Membership Table

- 2-2 Ex.35
- Show that $A \oplus B = (A \cup B) - (A \cap B)$

A	B	$A \oplus B$	$A \cup B$	$A \cap B$	$(A \cup B) - (A \cap B)$
0	0	0	0	0	0
0	1	1	1	0	1
1	0	1	1	0	1
1	1	0	1	1	0

- Do not be confused with truth table

2-3

Functions

2-3 Ex.6

- Find the domain and range of these functions
 - b) the function that assigns to each positive integer its largest decimal digit
 - c) the function that assigns to a bit string the number of ones minus the number of zeros in the string
 - e) the function that assigns to a bit string the longest string of ones in the string
- \mathbf{Z}^+ ; $\{1,2,3,4,5,6,7,8,9,\}$
- The set of bit strings; \mathbf{Z}
- The set of bit strings; the set of string of 1's: $\{\emptyset,1,11,111,\dots\}$

2-3 Ex.8

- Find these values:

– a) $\lfloor 1.1 \rfloor$ **1**

– b) $\lceil 1.1 \rceil$ **2**

– c) $\lfloor -0.1 \rfloor$ **-1**

– d) $\lceil -0.1 \rceil$ **0**

– e) $\lceil 2.99 \rceil$ **3**

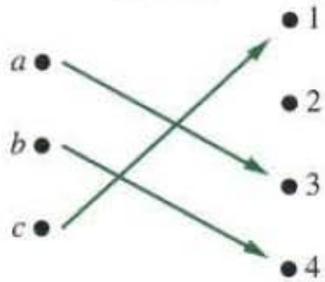
– f) $\lfloor -2.99 \rfloor$ **-2**

– g) $\left\lfloor \frac{1}{2} + \left\lceil \frac{1}{2} \right\rceil \right\rfloor$ **1**

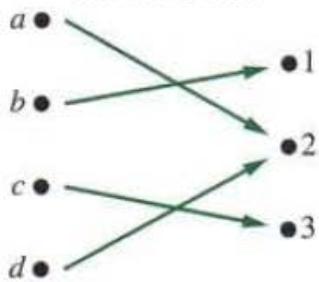
– h) $\left\lceil \left\lfloor \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor + \frac{1}{2} \right\rceil$ **2**

1-1 and Onto Functions

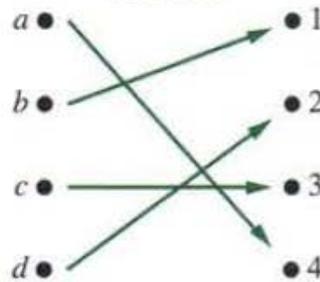
(a) One-to-one,
not onto



(b) Onto,
not one-to-one



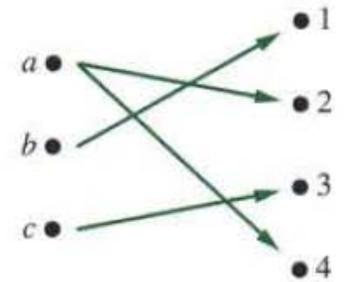
(c) One-to-one,
and onto



(d) Neither one-to-one
nor onto



(e) Not a function



2-3 Ex.12

- Determine whether each of these functions from \mathbf{Z} to \mathbf{Z} is one to one.

– a) $f(n) = n - 1$ Y

– b) $f(n) = n^2 + 1$ N, $f(3) = f(-3) = 10$

– c) $f(n) = n^3$ Y

– d) $f(n) = \lfloor n/2 \rfloor$ N, $f(3) = f(4) = 2$

2-3 Ex.14

- Determine whether $f: \mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z}$ is onto if
 - a) $f(m, n) = 2m - n$ **Y**
 - b) $f(m, n) = m^2 - n^2$ **N**
 - c) $f(m, n) = m + n + 1$ **Y**
 - d) $f(m, n) = |m| - |n|$ **Y**
 - e) $f(m, n) = m^2 - 4$ **N**

2-3 Ex.18

- Determine whether each of these functions is a bijection from \mathbf{R} to \mathbf{R}
 - a) $f(x) = -3x + 4$
 - b) $f(x) = -3x^2 + 7$
 - c) $f(x) = (x + 1)/(x + 2)$
 - d) $f(x) = x^5 + 1$
- $f^{-1}(x) = (4 - x)/3$
- not 1-1 since $f(17) = f(-17)$, and not onto since the range is $(-\infty, 7]$
- $f^{-1}(x) = (1 - 2x)/(x - 1)$, bijection, but not from \mathbf{R} to \mathbf{R}
- $f^{-1}(x) = \sqrt[5]{x - 1}$

2-3 Ex.34

- Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c , and d are constants. Determine for which constants a, b, c , and d it is true that $f \circ g = g \circ f$.
- $(f \circ g)(x) = acx + ad + b$
- $(g \circ f)(x) = cax + cb + d$
- $\rightarrow ad + b = cb + d$

2-3 Ex.68

- Suppose that f is a function from A to B , where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one iff it is onto.
- 1-1 \rightarrow onto:
 - if not onto, $|B|$ is at least one greater than $|A|$
- onto \rightarrow 1-1:
 - if not 1-1, $|A|$ is at least one greater than $|B|$

2-S Ex.13

- Let f and g be functions from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$ and from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ respectively, such that $f(1) = d$, $f(2) = c$, $f(3) = a$, $f(4) = b$ and $g(a) = 2$, $g(b) = 1$, $g(c) = 3$, $g(d) = 2$
 - a) Is f one-to-one? Is g one-to-one? **Y; N**
 - b) Is f onto? Is g onto? **Y; N**
 - c) Does either f or g have an inverse? **Y; N**

Floor and Ceiling Functions

$$\lfloor \frac{1}{2} \rfloor = 0, \lceil \frac{1}{2} \rceil = 1, \lfloor -\frac{1}{2} \rfloor = -1, \lceil -\frac{1}{2} \rceil = 0$$

$$\lfloor 3.1 \rfloor = 3, \lceil 3.1 \rceil = 4, \lfloor 7 \rfloor = 7, \lceil 7 \rceil = 7$$

TABLE 1 Useful Properties of the Floor and Ceiling Functions.

(n is an integer)

(1a) $\lfloor x \rfloor = n$ if and only if $n \leq x < n + 1$

(1b) $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$

(1c) $\lfloor x \rfloor = n$ if and only if $x - 1 < n \leq x$

(1d) $\lceil x \rceil = n$ if and only if $x \leq n < x + 1$

(2) $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$

(3a) $\lfloor -x \rfloor = -\lceil x \rceil$

(3b) $\lceil -x \rceil = -\lfloor x \rfloor$

(4a) $\lfloor x + n \rfloor = \lfloor x \rfloor + n$

(4b) $\lceil x + n \rceil = \lceil x \rceil + n$

2-3 Ex.54

- How many bytes are required to encode n bits of data where n equals
 - a) 4? $\lceil 4/8 \rceil = 1$
 - b) 10? $\lceil 10/8 \rceil = 2$
 - c) 500? $\lceil 500/8 \rceil = 63$
 - d) 3000? $\lceil 3000/8 \rceil = 375$

2-3 Ex.70 -c)

- Prove $\lceil \lceil x/2 \rceil / 2 \rceil = \lceil x/4 \rceil$ for all real number x
- Let $x = 4n + k$, where $0 \leq k < 4$
- if $k = 0 \rightarrow n = n$, true
- if $0 < k \leq 2$, then $\lceil x/2 \rceil = 2n + 1$, $\lceil (2n + 1)/2 \rceil = n + 1$
- if $2 < k < 4$, then $\lceil x/2 \rceil = 2n + 2$, $\lceil (2n + 2)/2 \rceil = n + 1$
- Since we proved all cases, the proof is complete

2-4

Sequences and Summations

2-4 Ex.8

- Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.
- 3, 5, 7, 9, 11, 13,
- 3, 5, 7, 11, 13, 17,
- Solve $y = Ax^3 + Bx^2 + Cx + D$ where (1, 3), (2, 5), (3, 7), (4, n) have been plugged in for x and y .

2-4 Ex.10

- For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.
 - a) 3, 6, 11, 18, 27, 38, 51, 66, 83, 102, ...
 - d) 1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, ...
 - e) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...
- $n^2 + 2$; 123, 146, 171
- for different value n , $n_k = n_{k-2} + n_{k-1}$; 8, 8, 8
- $n^3 - 1$; 59048, 177146, 531440

2-4 Ex.32

- Determine whether each of these sets is countable or uncountable. For those that are countable, exhibit a one-to-one correspondence between the set of natural numbers and that set.
 - a) the integers greater than 10
 - d) integers that are multiples of 10
- This set is countable; in general $n \leftrightarrow (n + 10)$.
- This set is countable; $1 \leftrightarrow 0, 2 \leftrightarrow 10, 3 \leftrightarrow -10, 4 \leftrightarrow 20, 5 \leftrightarrow -20, 6 \leftrightarrow 30$, and so on.